Shifting and splitting of resonance lines due to dynamical friction in plasmas

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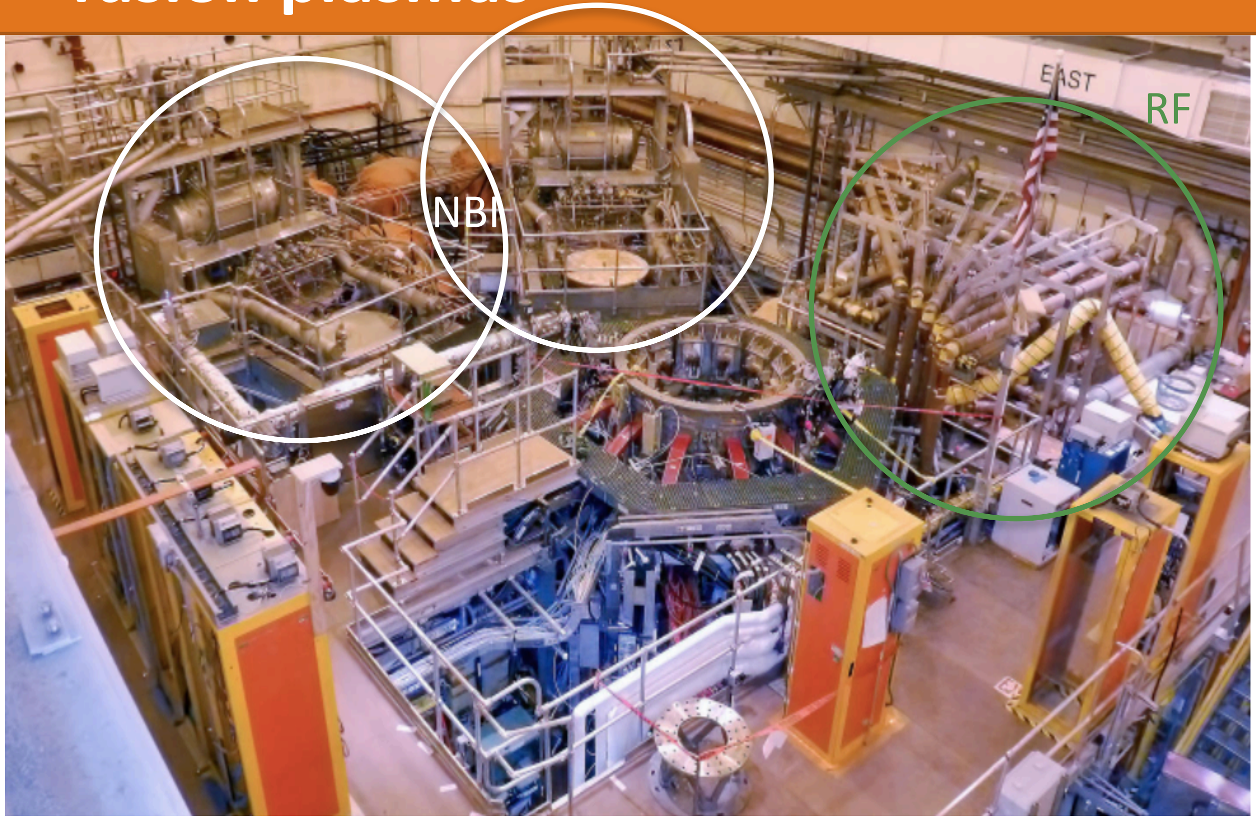
Instituto de Física, Universidade de São Paulo, May 12th 2022

A sub-population of energetic particles is ubiquitous in fusion plasmas

Sources of energetic particles:

- RF heating
- neutral beam injection
- alpha particles
- runaway electrons

Those energetic particles create an energy-inverted distribution that drive several instabilities, most notably Alfvénic modes. Control is necessary! A hierarchy of models is available…



NSTX-U, Princeton Plasma Physics Laboratory

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Quasilinear theory is a reduced approach to kinetic instabilities

In a regime where there is no effective particle trapping in resonances, the kinetic (Vlasov) description of phase mixing can be approximated by an irreversible, diffusive process

$$f\left(\varphi,\Omega,t\right)$$

$$\frac{\partial f}{\partial t}+\Omega\frac{\partial f}{\partial\varphi}+Re\left(\omega_b^2 e^{i\varphi}\right)\frac{\partial f}{\partial\Omega}=C\left[f,F_0\right]$$

$\longrightarrow$

$$f\left(\Omega,t\right)=\left\langle f\left(\varphi,\Omega,t\right)\right\rangle_\varphi$$

$$\frac{\partial f}{\partial t}-\frac{\partial}{\partial\Omega}D\frac{\partial f}{\partial\Omega}=C\left[f,F_0\right]$$

$$\dot{\varphi}=\partial H_0\left(J\right)/\partial J\equiv\Omega\left(J\right)$$

J represents $\left(\mathcal{E},P_\varphi,\mu\right)$

For quasilinear theory to be valid, the linear mode properties (e.g., eigenstructure and resonance condition) should not change in time

Quasilinear diffusion theory was independently proposed by
A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Sov. Phys. Usp. 4, 332 (1961).
W. Drummond and D. Pines, Nucl. Fusion Suppl. 2(Pt. 3), 1049 (1962).
Later generalized to action-angle variables:
A. N. Kaufman, Phys. Fluids 15, 1063 (1972).

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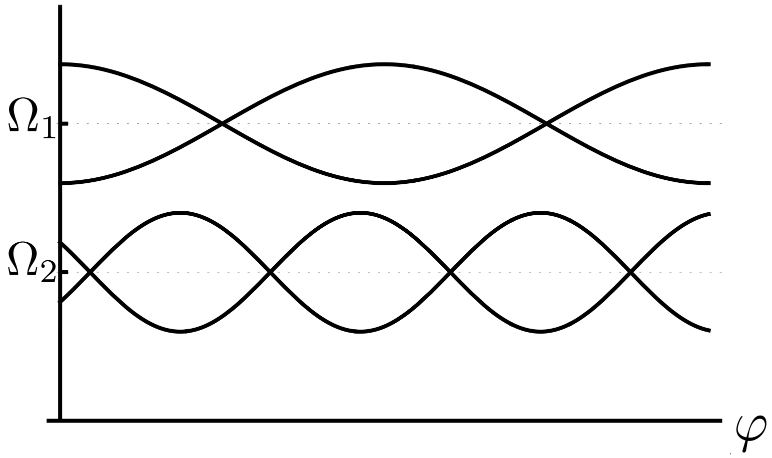
Historically, resonance overlap (Chirikov criterion) has been invoked to justify the applicability of QL theory

$$\omega_{b,1} + \omega_{b,2} \gtrsim |\Omega_1 - \Omega_2|$$

ω_b is the bounce (trapping) frequency



In this case, most trapped particles will not “belong” to a particular wave anymore but will be “shared” by the two waves.

- Intrinsic stochastic diffusion: due to interaction with broad spectrum
- Extrinsic stochasticity: by collisions inducing randomization of phase

The end goal of this talk is to show that in the presence of collisions, a QL theory can be formulated from first principles near marginal stability, even for a single resonance. Interesting properties emerge:

(i) it recovers the saturation level predicted by nonlinear theory
(ii) the resonance function can be analytically calculated

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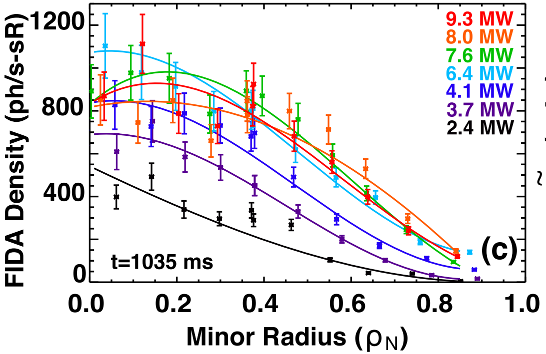
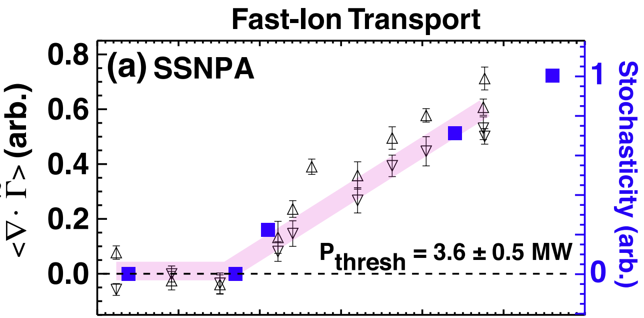
Critical gradient behavior in DIII-D suggests that quasilinear modeling is a viable modeling tool for fast ion relaxation

DIII-D critical gradient experiments

-stiff transport and resilient fast ion profiles as beam power varies

-stochastic fast ion transport (mediated by overlapping resonances) gives credence in using a quasilinear approach





- Fully nonlinear modeling of fast ion interaction with Alfvénic modes in a realistic tokamak is numerically expensive

- Simulations need to cope with the simultaneous excitation of multiple unstable Alfvénic instabilities, with and without overlap

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C. Collins et al, PRL 2016

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

Early development of broadened quasilinear theory

- The broadening of resonances is a ubiquitous phenomenon in physics (e.g., in atomic spectra)

- In plasma physics, broadened strong turbulence theories for dense spectra have been developed (e.g., Dupree, Phys. Fluids 1966);

The line broadening model ($\delta(\Omega) \to \mathcal{R}(\Omega)$):

$$\frac{\partial f(\Omega,t)}{\partial t} - \frac{\pi}{2}\frac{\partial}{\partial \Omega}\left[\left|\omega_b^2\right|^2 \mathcal{R}(\Omega)\frac{\partial f(\Omega,t)}{\partial \Omega}\right] = C[f,F_0]$$

$$d\left|\omega_b^2\right|^2/dt = 2\left(\gamma_L(t) - \gamma_d\right)\left|\omega_b^2\right|^2$$

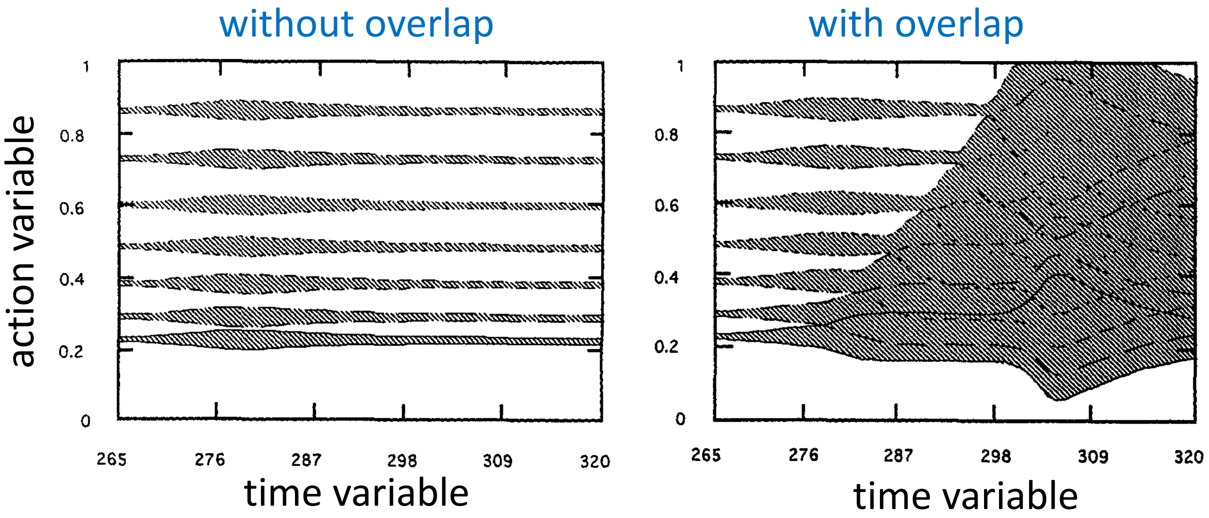
$$\gamma_L(t) = \tfrac{\pi}{4}\int_{-\infty}^{\infty} d\Omega \mathcal{R}\tfrac{\partial f(\Omega,t)}{\partial \Omega}$$

- $\mathcal{R}$ is an arbitrary resonance function (usually taken as in flat-top form) with $\int_{-\infty}^{\infty}\mathcal{R}(\Omega)d\Omega = 1$
- ω_b is the trapping (bounce) frequency at the elliptic point (proportional to square root of mode amplitude)

H. Berk, B. Breizman, J. Fitzpatrick, and H. Wong, Nucl. Fusion 35, 1661 (1995).

The overlapping of resonances lead to losses due to global diffusion

- The resonance broadened quasilinear model is designed to address both regimes of isolated and overlapping resonances
  - the fast ion distribution function relaxes while self-consistently evolving the amplitude of modes



H. Berk, B. Breizman, J. Fitzpatrick, and H. Wong, Nucl. Fusion 35, 1661 (1995).

First-principle analytical determination of the collisional resonance broadening – part I

Start with the kinetic equation: $\frac{\partial f}{\partial t}+\Omega\frac{\partial f}{\partial \varphi}+Re\left(\omega_b^2 e^{i\varphi}\right)\frac{\partial f}{\partial \Omega}=C\left[f,F_0\right]$ $\left\{\begin{array}{c}\nu_K\left(F_0-f\right)\\ \nu_{scatt}^3\partial^2\left(f-F_0\right)/\partial\Omega^2\\ \text{(from collisions, turbulence,...)}\end{array}\right.$

Periodicity over the canonical angle allows the distribution to be written as a Fourier series:

$$f\left(\varphi,\Omega,t\right)=F_0\left(\Omega\right)+\sum_{n=0}^{\infty}\left(f_n\left(\Omega,t\right)e^{in\varphi}+c.c.\right)$$

Near marginal stability, a perturbation theory can be developed in orders of $\omega_b^2/\nu_{K,scatt}^2$ which leads to the ordering $|F_0'|\gg\left|f_1'^{(1)}\right|\gg\left|f_0'^{(2)}\right|,\left|f_2'^{(2)}\right|$. When memory effects are weak, i.e., $\nu_{K,scatt}/\left(\gamma_{L,0}-\gamma_d\right)\gg 1$,

$$f_1=\frac{\omega_b^2F_0'}{2\left(i\Omega+\nu_K\right)}\qquad\qquad \frac{\partial f_0}{\partial t}+\frac{1}{2}\left(\omega_b^2\left[f_1'\right]^*+\omega_b^{2*}f_1'\right)=-\nu_K f_0$$

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First-principles analytical determination of the collisional resonance broadening – part II

When decoherence is strong, the distribution function has no angle dependence:

$$f\left(\Omega,t\right) \equiv F_0\left(\Omega\right) + f_0\left(\Omega,t\right)$$

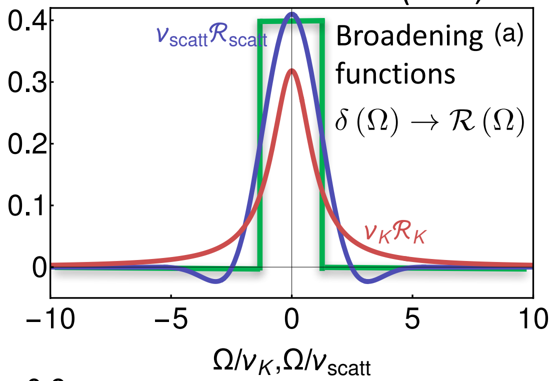
In the limit $\nu_{K,scatt}/\left(\gamma_{L,0}-\gamma_d\right) \gg 1$, the distribution relaxation is naturally cast as a diffusion equation:

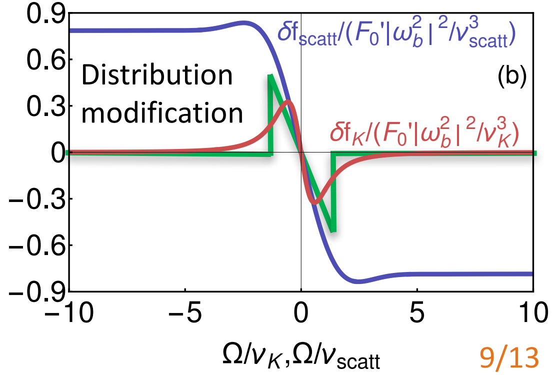
$$\frac{\partial f\left(\Omega,t\right)}{\partial t} - \frac{\pi}{2}\frac{\partial}{\partial \Omega}\left[\left|\omega_b^2\right|^2 \mathcal{R}\left(\Omega\right)\frac{\partial f\left(\Omega,t\right)}{\partial \Omega}\right] = C\left[f,F_0\right]$$

With the spontaneously emerged collisional resonance functions (both satisfy $\int_{-\infty}^{\infty}\mathcal{R}(\Omega)d\Omega = 1$):

$$\mathcal{R}_K\left(\Omega\right) = \frac{1}{\pi\nu_K\left(1+\Omega^2/\nu_K^2\right)} \qquad \mathcal{R}_{scatt}\left(\Omega\right) = \frac{1}{\pi\nu_{scatt}}\int_0^\infty ds\,\cos\left(\frac{\Omega s}{\nu_{scatt}}\right)e^{-s^3/3}$$

Blue curve: pitch-angle scattering
Red curve: Krook collisions
Green curve: previous heuristic broadening (Berk, NF '95)





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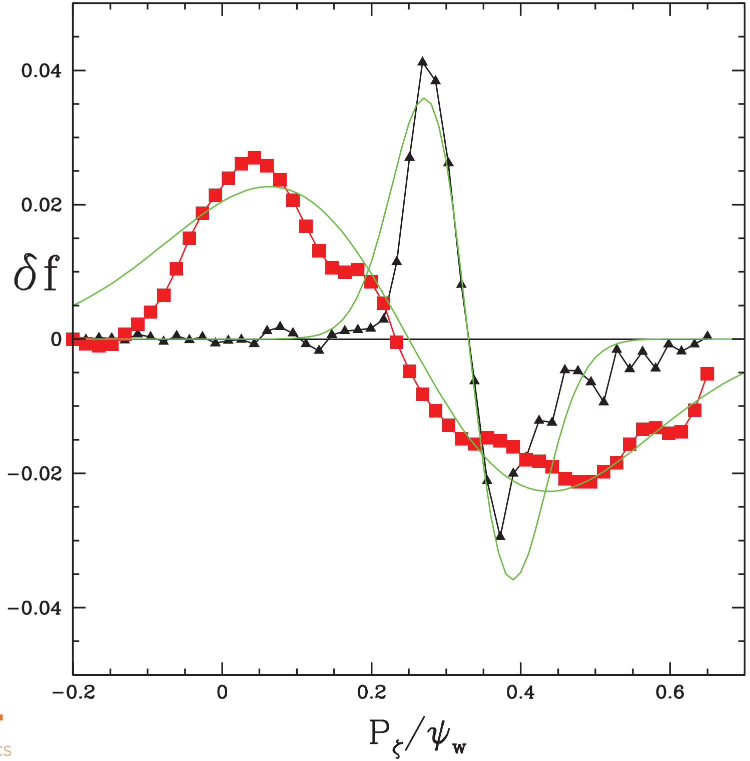
Verification of the analytical predictions against ORBIT simulations of Alfvénic resonances

Modification of the distribution function vs canonical toroidal momentum



Red and **black**: guiding-center ORBIT simulation results for two different levels of collisionality

Green: analytic fit

White, Duarte *et al*, *Phys. Plasmas* **26**, 032508 (2019)

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Self-consistent formulation of collisional quasilinear transport near threshold replicates essential features of nonlinear theory

$$\frac{\partial f(\Omega,t)}{\partial t}-\frac{\pi}{2}\frac{\partial}{\partial\Omega}\left[\left|\omega_b^2\right|^2\mathcal{R}(\Omega)\frac{\partial f(\Omega,t)}{\partial\Omega}\right]=C[f,F_0]$$

$$\gamma_L(t)=\frac{\pi}{4}\int_{-\infty}^{\infty}d\Omega\mathcal{R}\frac{\partial f(\Omega,t)}{\partial\Omega}$$

$$d\left|\omega_b^2\right|^2/dt=2\left(\gamma_L(t)-\gamma_d\right)\left|\omega_b^2\right|^2$$

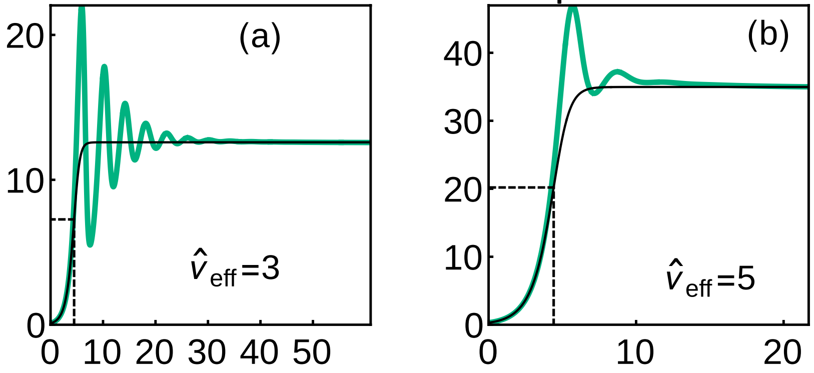
The QL plasma system automatically replicates the nonlinear growth rate and the wave saturation levels $|\omega_{b,sat}|=8^{1/4}\left(1-\gamma_d/\gamma_{L,0}\right)^{1/4}\nu_K$ calculated from fully kinetic theory near marginality,

$$\frac{d}{dt}\,\omega_B^2=(\gamma_L-\gamma_d)\omega_B^2(t)-\frac{\gamma_L}{2}\int_{t/2}^{t}dt'\,(t-t')^2\omega_B^2(t')\times$$
$$\times\int_{t-t'}^{t'}dt_1\,\exp[-\nu(2t-t'-t_1)]\,\omega_B^2(t_1)\omega_B^2(t'+t_1-t)$$

(Berk, Breizman and Pekker, *Phys. Rev. Lett.* 1996)

Amplitude vs time

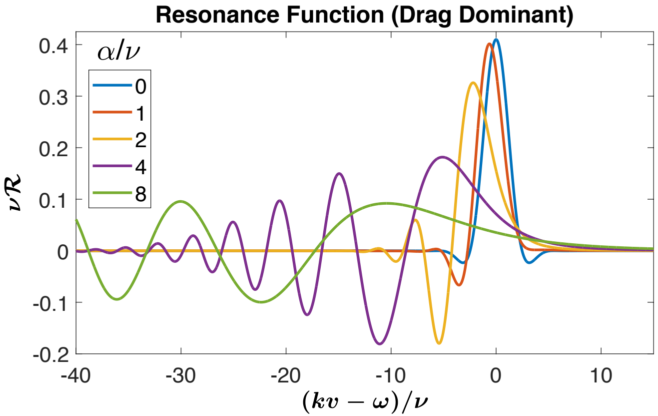
Green: nonlinear **Black**: quasilinear

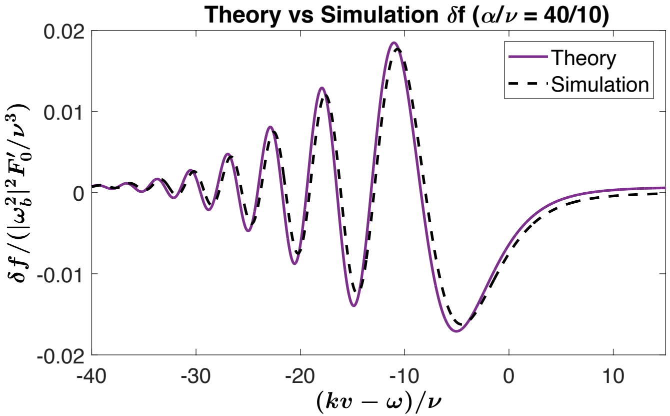


(Duarte & Gorelenkov, *Nucl. Fusion* 2019)

Collisional slowing down (drag) leads to shifting and splitting of resonances

$$\frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \varphi} + Re\left(\omega_b^2 e^{i\varphi}\right) \frac{\partial f}{\partial \Omega} = \nu^3 \frac{\partial^2 (f - F_0)}{\partial \Omega^2} + \alpha^2 \frac{\partial (f - F_0)}{\partial \Omega}$$





- Agreement between quasilinear and nonlinear theories on the resonant particle redistribution
- Previously unrecognized universality of resonance behavior present in kinetic plasma physics

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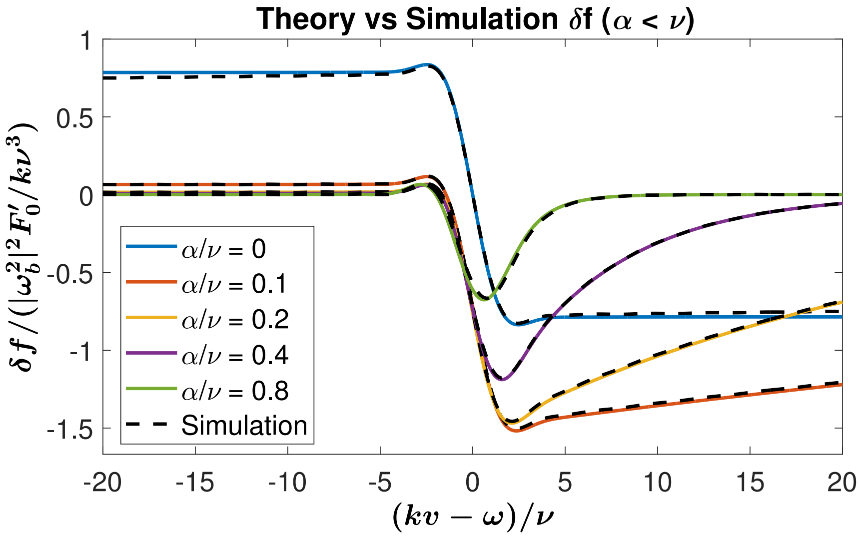
Duarte, *et al*, "*Shifting and splitting of resonance lines due to dynamical friction in plasma*", (submitted, 2021)

Drag breaks the anti-symmetry of the relaxed distribution around a resonance

- The amount of shifting and splitting is dictated by the effective drag to scattering ratio:

$$\frac{\alpha}{\nu} \approx \left(0.43\frac{\mathrm{Hz}^{1/6}\mathrm{keV}^{1/4}}{(10^{20}\mathrm{m}^{-3})^{1/6}}\right)\frac{\mathcal{E}^{1/2}n_e^{1/6}}{T_e^{3/4}\omega^{1/6}}$$

- In conventional tokamaks, the ratio is 0.1-0.5.
- Spherical tokamaks, the ratio is ~1.
- Basic plasma physics experiments (LAPD, TORPEX,…), the ratio is ~10.



Any small amount of drag breaks structural constraints of the system and allows for the loss of resonant particles via new channels -> drag needs to be included for quantitative predictions

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Duarte, *et al*, *“Shifting and splitting of resonance lines due to dynamical friction in plasma”*, (submitted, 2021)

Summary

- A systematic QL theory has been derived from first principles near an instability threshold, where the collisional resonance functions emerge spontaneously:
  - Scattering broadens the resonance function
  - Drag leads to shifting and splitting of the resonance function

- The derivation indicates that QL theory can be applicable to a single discrete resonance (with no overlap), provided that stochasticity is large enough, as well as the usual overlapping regime

- An arbitrariness of collisional QL modeling (the shape of the resonance functions) has been removed

- The QL system (with the calculated broadening functions) systematically recovers the mode saturation levels for near-threshold plasmas previously calculated from nonlinear kinetic theory

- Resonance functions have been implemented into the Resonance Broadening Quasilinear (RBQ) code

The use of the obtained resonance functions implies that fundamental features of nonlinear theory are automatically built into broadened QL theory

Duarte, *et al*, "*Collisional resonance function in discrete-resonance quasilinear plasma systems*", *Phys. Plasmas* **26**, 120701 (2019)
Duarte, *et al*, "*Shifting and splitting of resonance lines due to dynamical friction in plasma*", (submitted, 2021)

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Obrigado!

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